Quantitative Analysis

The Very Basics

David Chiang
October 2023

Wikipedia Definitions

- **Quantitative analysis** - the use of mathematical and statistical methods in finance and investment management
- **Return** (rate of return) - a profit on an investment over a period of time
- **Volatility** - measure of the amount of variation of a set of values
- **Skewness** - measure of the asymmetry of the probability distribution
- **Kurtosis** - measure of the "tailedness" of the probability distribution of a real-valued random variable
- **Sharpe Ratio** – measure of the performance of an investment compared to a risk-free asset, after adjusting for its risk
- **Back Testing** - testing a predictive model on historical data

Can Quantitative Analyses Predict the Future

- Yes
  - Quantitative analyses provide the parameters that specify the statistical behavior of future financial data
- No
  - Financial data are non-stationary (statistics change over time)

Returns

- Returns over a time period
  - Discrete = $(p_n - p_{n-1}) / p_{n-1}$
  - Continuous = $\log(p_n / p_{n-1})$
- Approximate historical returns using a normal distribution
- Expected return is the average of the distribution
  - E.g. Annual return the expected return over a year
  - May be calculated by taking average of many years or
  - Annualize average monthly returns

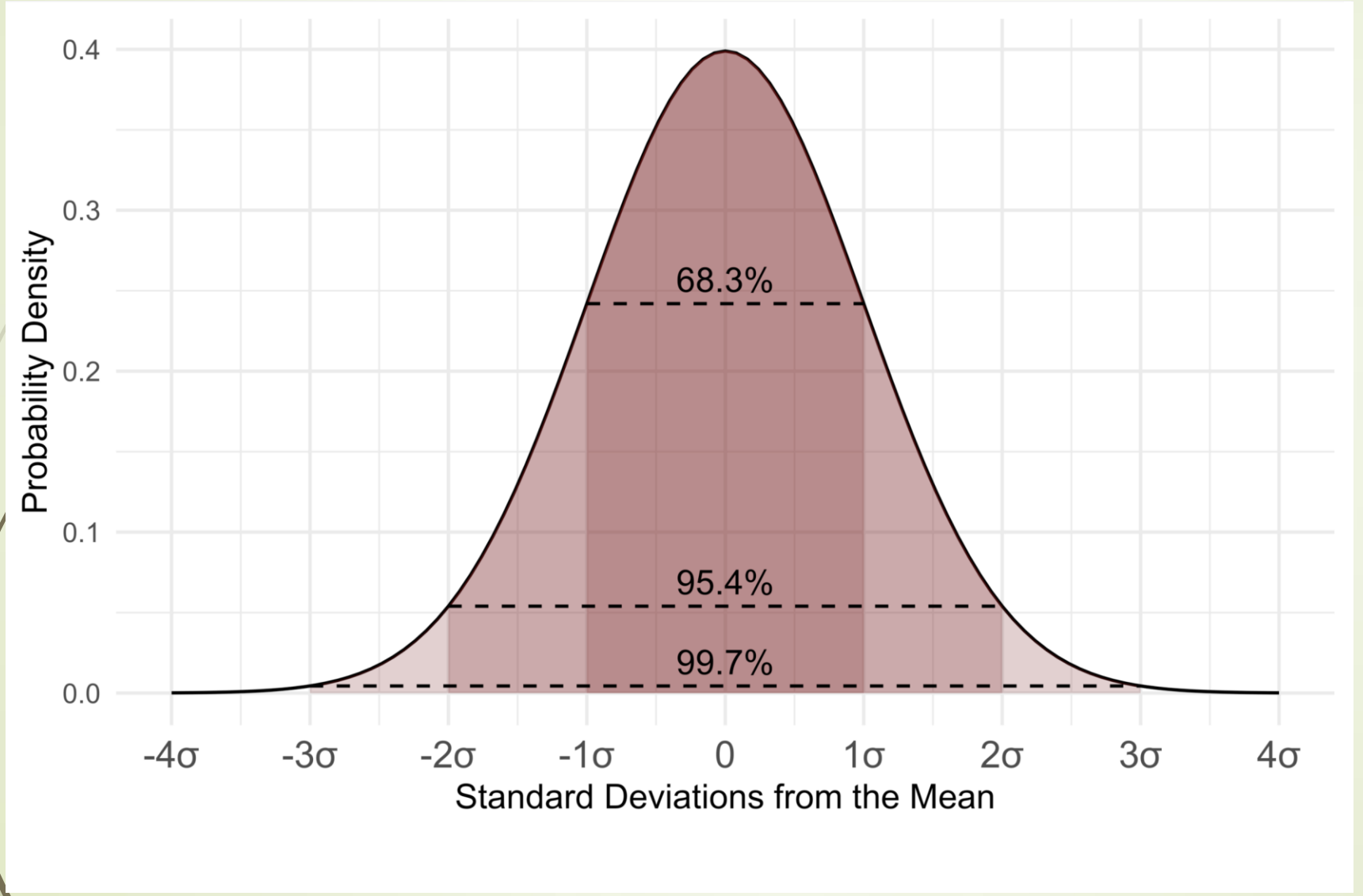
** SW used to demonstrate returns is derived from Reproducible Finance with R by Jonathan K. Regenstein

Volatility (Risk)

- Standard deviation is typically used for volatility or risk
- Standard deviation
  - Denoted by σ
  - Calculated over a period of time (e.g. Per Annum, quarterly, monthly, etc.)
- Another commonly used measure for risk is drawdown
  - Largest reduction from highest value to lowest
  - Normally used in back testing

** SW used to calculate standard deviations is derived from Reproducible Finance with R by Jonathan K. Regenstein

Normal Distribution



1-2 σ
(95.4%-68.3%)/2 = 27.1%/2 = 13.55%

2-3 σ
(99.7% -95.4%)/2 = 4.3%/2 = 2.15%

What Does 10% Volatility Mean?

- 10% per annum ~ 0.63% per day (AAPL volatility)
  - .1/ SQRT(250)=.1/15.81=.0063
- Number of days per year of 1-2 sd drop [-.63% to -1.26%]
  - 250 * (95-68)/2=250 * 13.5%= 34 trading days (or 1.5 months)
- Probability of having 3 down days of 1 sd or more (ie down 1.8% or more)
  - (13.55% + 2.15% + 0.15%)**3= 0.4%
  - The probability is in fact a lot higher

Skewness and Kurtosis

- Return distributions are typically skewed to the high or low side
- Return distributions are "Fat tailed" – a lot more occurrences at the extreme values than normal distribution

** SW used to demonstrate skewness and Kurtosis is derived from Reproducible Finance with R by Jonathan K. Regenstein

Trade-off Between Risk and Reward

- What is better?
  - Higher return with high risk
  - Lower return with lower risk
- How does quantitative analysis address this question?

Sharpe Ratio

- Introduced by Nobel Laureate William Sharpe

$$\text{Sharpe Ratio} = \text{Average}(R - R_f)/\sigma$$

Rf is the risk free rate; one typically uses US treasury rate

- Provides a metric to evaluate assets with different return and risk profiles

** SW used to demonstrate Sharpe Ratio is derived from Reproducible Finance with R by Jonathan K. Regenstein

Optimal Portfolio

- The optimal portfolio of assets is the one having the set of weights that minimizes the portfolio standard deviation (Mean-Variance Optimization)
- Mathematics to find the “optimal” weights is messy
- For a portfolio of two assets, one can find the optimal weights by brute force

Back Testing

- Assume that you have an investment strategy
  - How much return should you expect in the future?
  - How did it perform in the past?
- Back testing provides
  - The return you would have received had you implemented the strategy
  - The risks you would have suffered in the process
- No guarantee that good historical performance will repeat, but definitely better than
  - Irrational exuberance
  - Excessive pessimism

That’s all folks
Thank you for attending
Questions?